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Influence of weighted function exponent in WFEM on error of solution for hydrodynamic problems with singularity

The concept of an R_ν-generalized solution for a hydrodynamic problem with reentrant corner on the boundary of a polygonal domain is defined. An approximate method for solving the problem is constructed. A numerical analysis is carried out and the question of the influence of the weighted function exponent in the weighted finite element method on the error of the solution in the vicinity of the reentrant corner in the norm of the space $\mathbf{C}(\bar{\Omega})$ is experimentally studied. A comparative analysis has been carried out and the advantage of the weighted method over the classical approach has been shown.

Key words: *Navier-Stokes equations, weighted FEM, corner singularity.*

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Introduction

At present, mathematicians and engineers are of particular interest in solving problems describing natural physical processes in polygonal non-convex domains Ω with a corner ω on the boundary greater than π. There is a task class for which a generalized solution exists and belongs to the space $W_2^1(\Omega)$, but it does not belong to the Sobolev space $W_2^2(\Omega)$. According to the principle of agree estimates there is no classical finite difference method (CFDM) or finite element method (CFEM), whose solution would converge to the exact one at the rate of $\mathcal{O}(h)$(where h is the grid step). In reality, the order of convergence with respect to h is substantially less than unity and decreases with increasing ω. As for the hydrodynamic problems, we single out the following approaches to research based on: 1) the separation of singular and regular components of the solution, approximation of the coefficients of the 1st and, based on this, finding the 2nd [1]; 2) mesh thickening in the vicinity of ω [2]. Known methods allow obtain the required 1st order of convergence, but in the $\mathcal{O}(1)$-vicinity of ω modifications to the CFEM are required.

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This paper proposes a fundamentally different idea. The solution is defined as R_ν-generalized in weight sets. The approach is based on the introduction of the weight function into the integral identities, equal to the distance from the point to the singularity point in the δ-neighbourhood of the reentrant corner, and the constants δ outside it. Thanks to this, the influence of the corner singularity can be suppressed. This idea was first proposed for solving elliptic problems (see [3]). For the Stokes problem in [4] a weight analogue of the LBB condition is established. The existence and uniqueness of an R_ν-generalized solution in weighted sets [5] is proved. Numerical methods proposed for the elliptic problems [6–8] with different values of corners ω. The paper considers the stationary problem flow of a homogeneous incompressible viscous fluid, obtained as a result of discretization in time of the Navier–Stokes equations describing it in convective form. For the numerical implementation of the problem, based on the R_ν-generalized solution, a weighted FEM (WFEM) is constructed. A series of computational experiments was carried out for an angle greater than π, as with the help of the classical FEM, and the proposed WFEM. Comparative analysis is carried out in the norm of the space $\mathbf{C}(\bar{\Omega})$.

1 Problem statement and definition of R_ν-generalized solution

Let $\Omega = \{\mathbf{x} : \mathbf{x} = (x_1, x_2)\} \subset \mathbf{R}^2$ be a polygonal domain with boundary $\Gamma, \bar{\Omega} = \Omega \cup \Gamma$. Consider a nonlinear problem: find the velocity $\mathbf{w} = (w_1, w_2)$ and the pressure q, that is, the solution of the system and boundary condition

$$\alpha\mathbf{w} - \nabla \cdot (\bar{\nu}\nabla\mathbf{w}) + (\mathbf{w} \cdot \nabla)\mathbf{w} + \nabla q = \mathbf{f}, \quad \nabla \cdot \mathbf{w} = 0 \quad \text{in} \quad \Omega, \quad \mathbf{w} = \mathbf{g} \quad \text{on} \quad \Gamma, \tag{1}$$

where $\alpha, \bar{\nu}$ known positive constant, $\mathbf{f}$ and $\mathbf{g}$ known forces on Ω and Γ respectively. Problem (1) is nonlinear due to the presence of the term $(\mathbf{w}\cdot\nabla)\mathbf{w}$. The most appropriate way to linearized it is Picard's iterative procedure [9]: at the n-th iteration, we solve the problem with the linear term $(\mathbf{w}^{n-1} \cdot \nabla)\mathbf{w}^n$, where $\mathbf{w}^{n-1}$ computed in the previous iteration. If the norm of the $\mathbf{f}$ is bounded and $\bar{\nu}$ is not so small, then there exists a solution (1) $(\mathbf{w}, q)$ and $\mathbf{w}^n \to \mathbf{w}, q^n \to q$ for $n \to \infty$ for any initial approximation $\mathbf{w}^0$ satisfying (1). Thus, it is necessary to solve the following problem: find $\mathbf{u} = (u_1, u_2)$ and p, satisfying the system of equations and the boundary condition:

$$\alpha\mathbf{u} - \nabla \cdot (\bar{\nu}\nabla\mathbf{u}) + (\mathbf{d}^n \cdot \nabla)\mathbf{u} + \nabla p = \mathbf{f}, \quad \nabla \cdot \mathbf{u} = 0 \quad \text{in} \quad \Omega, \quad \mathbf{u} = \mathbf{g} \quad \text{on} \quad \Gamma, \tag{2}$$

where $\mathbf{d}^n \in \mathbf{L}_\infty(\Omega)$ is an approximate of the velocity from previous iteration, $\nabla \cdot \mathbf{d}^n = 0$.

A feature of the consideration of problem (1) is that the domain Ω is a non-convex polygon with one corner ω with vertex at the origin $(0, 0)$ the value of which on the boundary is greater than π. Thus, in accordance with the principle of consistent estimates, any FEM cannot give a better result in order of accuracy of the solution (see [10]) than

$$\|\nabla(\mathbf{u} - \mathbf{u}_h)\|_{\mathbf{L}_2(\Omega_h)} + h^{-\beta}\|\mathbf{u} - \mathbf{u}_h\|_{\mathbf{L}_2(\Omega_h)} = \mathcal{O}(h^\beta).$$

For example, if ω takes a value $\frac{3\pi}{2}$, then $\beta \approx 0.54$. In a convex domain β equals to 1. Now we define an R_ν-generalized solution to the problem (2). The idea of an R_ν-generalized solution is based on the introduction of the weight function $\rho(\mathbf{x})$ into the integral identity to some positive power ν, and

$$\rho(\mathbf{x}) = \{\|\mathbf{x}\|,\ \text{if } \mathbf{x} \in \Omega_\delta; \delta,\ \text{if } \mathbf{x} \in \bar{\Omega}\backslash\Omega_\delta\},$$

where $\Omega_\delta = \{\mathbf{x} \in \bar{\Omega} : \|\mathbf{x}\| \le \delta \ll 1\}$. Now, we introduce the necessary sets of generalized functions (see [11–13]). Denote by $W^1_{2,\eta}(\Omega,\delta)$ the set of functions $z(\mathbf{x})$, satisfying the conditions:

$$0 < C_1 \le \|z\|_{L_{2,\eta}(\Omega\backslash\Omega_\delta)}, |D^k z(\mathbf{x})| \le C_2 \delta^{\eta-\tau} \rho^{\tau-\eta-k}(\mathbf{x}), \mathbf{x} \in \Omega_\delta, \tag{3}$$

and having a limited norm

$$\|z\|_{W^1_{2,\eta}(\Omega)} := \sqrt{\sum_{|l|\le 1} \|\rho^\eta |D^l z|\|^2_{L_2(\Omega)}}$$

of $W^1_{2,\eta}(\Omega)$, where C_2 is a positive constant and τ is a small positive parameter independent from η, δ and $z(\mathbf{x}), k = 0, 1$. Let $W^{1,0}_{2,\eta}(\Omega,\delta) \subset W^1_{2,\eta}(\Omega,\delta)$ be such that

$$W^{1,0}_{2,\eta}(\Omega,\delta) = \{z \in C^\infty(\bar{\Omega}) : z = 0 \text{ on } \Gamma,\ \text{satisfies conditions (3)}\}$$

with bounded norm of $W^1_{2,\eta}(\Omega)$.

$$W^{1/2}_{2,\eta}(\Gamma,\delta) = \{s :\ \text{if exists } z \in W^1_{2,\eta}(\Omega,\delta),\ \text{that } z = s \text{ on } \Gamma\}$$

with a

$$\|s\|_{W^{1/2}_{2,\eta}(\Gamma,\delta)} := \inf_{z=s \text{ on } \Gamma} \|z\|_{W^1_{2,\eta}(\Omega)}.$$

Via $L_{2,\eta}(\Omega,\delta)$ denote the set of functions $z(\mathbf{x})$ satisfying (3) if $k = 0$, with bounded norm $\|z\|_{L_{2,\eta}(\Omega)} := \|\rho^\eta z\|_{L_2(\Omega)}$ of the space $L_{2,\eta}(\Omega)$. $L^0_{2,\eta}(\Omega,\delta)$ is a subset of $L_{2,\eta}(\Omega,\delta)$ such that $z \in L^0_{2,\eta}(\Omega,\delta)$ if and only if $z \in L_{2,\eta}(\Omega,\delta)$ and $\|\rho^\eta z\|_{L_1(\Omega)} = 0$. We will highlight the spaces of vector functions in bold type.

Definition 1. The pair $(\mathbf{u}_\nu, p_\nu) \in \mathbf{W}^1_{2,\nu}(\Omega,\delta) \times L^0_{2,\nu}(\Omega,\delta)$ is called an R_ν-generalized solution of (2), $\mathbf{u}_\nu$ satisfies a condition (2) almost everywhere on Γ, if

$$a_n(\mathbf{u}_\nu, \mathbf{v}) - b_1(\mathbf{v}, p_\nu) = l(\mathbf{v}), \qquad b_2(\mathbf{u}_\nu, s) = 0 \tag{4}$$

hold, $\forall(\mathbf{v}, s) \in \mathbf{W}^{1,0}_{2,\nu}(\Omega,\delta) \times L^0_{2,\nu}(\Omega,\delta)$, $\mathbf{f} \in \mathbf{L}_{2,\zeta}(\Omega,\delta), \mathbf{g} \in \mathbf{W}^{1/2}_{2,\zeta}(\Gamma,\delta)$, $\nu \ge \zeta \ge 0:$

$$a_n(\mathbf{u}_\nu, \mathbf{v}) = \int_\Omega \Big[\alpha \mathbf{u}_\nu \cdot (\rho^{2\nu}\mathbf{v}) + \bar{\nu}\nabla\mathbf{u}_\nu : \nabla(\rho^{2\nu}\mathbf{v}) + (\mathbf{d}^n \cdot \nabla)\mathbf{u}_\nu \cdot (\rho^{2\nu}\mathbf{v})\Big] d\mathbf{x},$$

$$l(\mathbf{v}) = \int_\Omega \mathbf{f} \cdot (\rho^{2\nu}\mathbf{v}) d\mathbf{x}, b_1(\mathbf{v}, p_\nu) = \int_\Omega p_\nu \nabla \cdot (\rho^{2\nu}\mathbf{v}) d\mathbf{x}, b_2(\mathbf{u}_\nu, s) = \int_\Omega (\rho^{2\nu}\, s)\, \nabla \cdot \mathbf{u}_\nu \, d\mathbf{x}.$$

Table 1: WFEM ($\delta = 0.01265$ and $\nu = 1.6$) and CFEM (GS).

ν^*	0.24	0.26	0.28	0.3	0.32	0.34	0.36	GS
$(\Delta, h) = (10^{-6}, 0.005)$	0.14	0.18	0.30	0.25	0.19	0.15	0.12	0.11
$(\Delta, h) = (5 \cdot 10^{-7}, 0.005)$	0.09	0.12	0.21	0.20	0.13	0.10	0.08	0.07
$(\Delta, h) = (10^{-6}, 0.0025)$	0.30	0.44	0.86	0.47	0.31	0.24	0.20	0.17
$(\Delta, h) = (5 \cdot 10^{-7}, 0.0025)$	0.19	0.29	0.73	0.32	0.20	0.16	0.13	0.11

2 Construction of an approximate method for the problem

Let us construct a WFEM to find an approximate solution of (2), relying on the notion of an R_ν-generalized solution (4). For this purpose, we triangulate T_h of Ω. To do this, we divide Ω by triangles with sides of order h. We represent each of them as three triangles K_{i_j} with vertices in the center masses of the main triangle. Each of K_{i_j} will be called a FE, and $\Omega_h = \bigcup_{K_{i_j} \in T_h} K_{i_j}$. Further, we will find sets of nodes for the velocity and pressure. For the velocity, the set of FE vertices R_l and the midpoints of their sides Q_k, lying inside Ω will be denoted by S_Ω, and on the boundary by S_Γ, $S = S_\Omega \cup S_\Gamma$. The common vertex (or the middle of the side) of two adjacent FEs is considered to be one node. For the pressure, let's denote by Z the set of nodes Z_s coinciding with the nodes R_l corresponding K_i. Z_s and Z_j coinciding with the node R_l, the vertices of two neighboring elements are considered different nodes. For the velocity, to each node P_k of S we assign a basis function $\varphi_k(\mathbf{x}) : \varphi_k(P_k) = 1$ and $\varphi_k(P_j) = 0, j \neq k$, $\varphi_k(\mathbf{x})$ is a quadratic function on yours support. We define the space X^h as a linear span, spanned by a system of $\{\varphi_k\}_{k=1}^{|S|}$. For the velocity field, we will use the space $\mathbf{X}^h = X^h \times X^h$. For the pressure, each node Z_l of Z is associated with a basis function $\psi_l(\mathbf{x}) : \psi_l(Z_l) = 1$ and $\psi_l(Z_j) = 0, j \neq l$, $\psi_l(\mathbf{x})$ is a linear function on one finite element. We define the space Y^h as a linear span, spanned by the system of basis functions $\{\psi_l\}_{l=1}^{|Z|}$. Let us improve the pair of spaces $\mathbf{X}^h - Y^h$ as follows: 1) each $\varphi_k(\mathbf{x})$ of the space X^h multiply by $\rho^{-\nu^*}(\mathbf{x})$: $\chi_k(\mathbf{x}) := \rho^{-\nu^*}(\mathbf{x})\varphi_k(\mathbf{x})$, and multiply each $\psi_l(\mathbf{x})$ of the space Y^h by $\rho^{-\mu^*}(\mathbf{x}) : \theta_l(\mathbf{x}) := \rho^{-\mu^*}(\mathbf{x})\psi_l(\mathbf{x})$; 2) their linear spans form FE spaces W^h and Q^h, respectively. For the velocity field, we have $\mathbf{W}^h = W^h \times W^h$. Also $\mathbf{W}_0^h = \{\mathbf{v}^h \in \mathbf{W}^h : \mathbf{v}^h(P_k) = \mathbf{0},$ in all $P_k \in S_\Gamma\}$. Velocity field $\mathbf{u}_\nu^h = (u_{\nu,1}^h, u_{\nu,2}^h)$ and pressure p_ν^h have the form:

$$u_{\nu,i}^h(\mathbf{x}) = \sum_{k=1}^{|S|} c_k^i \chi_k(\mathbf{x}), i = 1, 2, \quad p_\nu^h(\mathbf{x}) = \sum_{l=1}^{|Z|} d_l \theta_l(\mathbf{x}).$$

The coefficients $c_k^i, i = 1, 2$, and d_l are found by solving the SLAE, obtained by the Galerkin method from (5) (see below). Then we find the true values of the velocity and pressure at the nodes of the S_Ω and Z sets, respectively, using the relations

$$u_{i,k} = \rho^{-\nu^*}(P_k) c_k^i, \ i = 1, 2, \quad p_l = \rho^{-\mu^*}(Z_l) d_l.$$

We use the case $\mu^* = \nu^*$ (see [14, 15]).

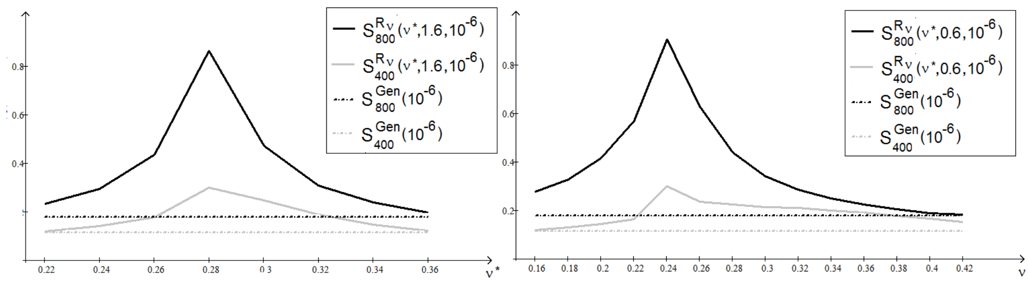

Fig. 1: WFEM for $\nu = 1.6$ (left), $\nu = 0.6$ (right).

Table 2: The WFEM ($\delta = 0.01265$ and $\nu = 0.6$) and CFEM.

ν^*	0.2	0.24	0.28	0.32	0.36	0.4	0.42	GS
$(\Delta, h) = (10^{-6}, 0.005)$	0.15	0.30	0.23	0.21	0.20	0.17	0.15	0.11
$(\Delta, h) = (5 \cdot 10^{-7}, 0.005)$	0.10	0.19	0.16	0.15	0.14	0.11	0.10	0.07
$(\Delta, h) = (10^{-6}, 0.0025)$	0.42	0.91	0.44	0.29	0.23	0.19	0.18	0.17
$(\Delta, h) = (5 \cdot 10^{-7}, 0.0025)$	0.28	0.80	0.29	0.19	0.17	0.13	0.12	0.11

Definition 2. The pair $(\mathbf{u}_\nu^h, p_\nu^h) \in \mathbf{W}^h \times Q^h$ is called an approximate R_ν-generalized solution of the problem (2), $\mathbf{u}_\nu^h$ satisfies a condition (2) at the nodes S_Γ, if

$$a_n(\mathbf{u}_\nu^h, \mathbf{v}^h) + b_1(\mathbf{v}^h, p_\nu^h) = l(\mathbf{v}^h), \qquad b_2(\mathbf{u}_\nu^h, s^h) = 0 \tag{5}$$

hold, $\forall(\mathbf{v}^h, s^h) \in \mathbf{W}_0^h \times Q^h$ and $\mathbf{f} \in \mathbf{L}_{2,\zeta}(\Omega, \delta), \mathbf{g} \in \mathbf{W}_{2,\zeta}^{1/2}(\Gamma, \delta)$, $\nu \geq \zeta \geq 0$.

3 Results of numerical experiments

Let us carry out numerical simulation of the nonlinear problem (1). The method for finding solution of (5) is based on the incomplete Uzawa algorithm [16]. Let $\Omega := (-1, 1) \times (-1, 1) \setminus [0, 1] \times [-1, 0]$. and define the triangulation T_h. Straight lines $x_1^{(i)} = -1 + ih, x_2^{(j)} = -1 + jh, i, j = 0, \dots, N$ for a given $N : N \cdot h = 2$, when intersecting with the domain $\bar{\Omega}$ split it into elementary squares. Each square with a diagonal (we connect its lower left vertex with the upper right vertex) divide into two triangles. Each of the resulting triangles, using the centroid, is divided into three triangles. The solution $(\mathbf{w}, q)$ of (1) has the form: $w_1(r, \varphi) = r^\lambda \Upsilon_1(\varphi), w_2(r, \varphi) = r^\lambda \Upsilon_2(\varphi), q(r, \varphi) = r^{\lambda-1}\Upsilon_3(\varphi)$. We have $\Upsilon_1(\varphi) = (1+\lambda)G(\varphi)\sin\varphi + G'_\varphi(\varphi)\cos\varphi, \Upsilon_2(\varphi) = G'_\varphi(\varphi)\sin\varphi - (1+\lambda)G(\varphi)\sin\varphi, \Upsilon_3(\varphi) = \frac{(1+\lambda)^2}{\lambda-1}G'_\varphi(\varphi) - \frac{1}{1-\lambda}G'''_{\varphi\varphi\varphi}(\varphi)$, where the auxiliary function is represented as $G(\varphi) := \frac{\sin((1+\lambda)\varphi)\cos(\frac{3\pi\lambda}{2})}{1+\lambda} + \frac{\sin((\lambda-1)\varphi)\cos(\frac{3\pi\lambda}{2})}{1-\lambda} + \cos((1-\lambda)\varphi) - \cos((1-\lambda)\varphi)$.

We use grids with a step h equal to $0.01, 0.005, 0.0025$ and $\alpha = 0.01, \bar{\nu} = 1, \lambda = 0.545$. Let's present comparative analysis of the CFEM, i.e. $\nu = \nu^* = \mu^* = 0$ and proposed WFEM, which has three parameters: δ, ν and $\nu^*(\mu^* = \nu^*)$ in the norm of $C(\bar{\Omega})$. We have determined the range of choice of the optimal parameter ν^* for $\delta = 0.01265$ and

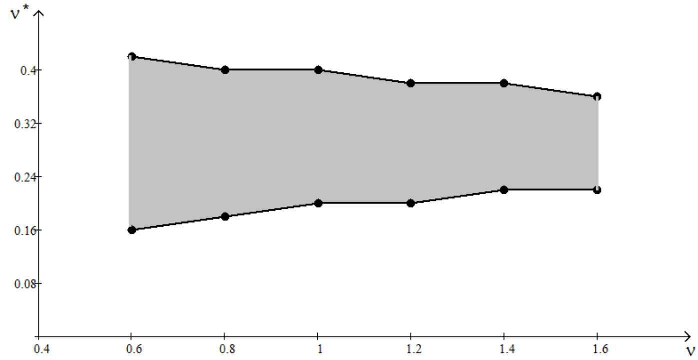

Fig. 2: Optimal parameter ν^* of WFEM, $\nu \in [0.6, 1.6]$ in the norm of $C(\bar{\Omega})$.

$\nu \in [0,6; 1.6]$. In Tables 1-2, we have determined the proportion of nodes where the error doesn't exceed Δ, which are $5 \cdot 10^{-7}$ and 10^{-6}. Figures 1-2 show graphs of the distribution of nodes, depending on ν^*, where $S_N^{Gen}(\Delta)$ – proportion of CFEM nodes not exceeding Δ and $S_N^{R_\nu}(\nu^*, \nu, \Delta)$ – proportion of WFEM nodes not exceeding Δ for $\delta = 0.01265$.

The proposed WFEM gives a significant advantage over the CFEM (see Figures 1-2, Tables 1-2). The error in the norm of the space $C(\bar{\Omega})$ is suppressed in the vicinity of the singularity point and does not allow it to propagate into the interior part of domain.

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АННОТАЦИЯ

Определено понятие R_ν-обобщённого решения для одной задачи гидродинамики с входящим углом на границе многоугольной области. Построен приближённый метод решения задачи. Проведён численный анализ и экспериментально изучен вопрос влияния показателя весовой функции весового метода конечных элементов на погрешность решения в окрестности входящего угла в норме пространства $C(\bar{\Omega})$. Выполнен сравнительный анализ и показано преимущество весового метода над классическими подходами.

Ключевые слова: *уравнения Навье – Стокса, ВМКЭ, угловая сингулярность.*