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Lie derivations on the algebra of measurable operators affiliated with a type I finite von Neumann algebra

Let M be a type I finite von Neumann algebra and let $S(M)$ be the algebra of all measurable operators affiliated with M. We prove that every Lie derivation on $S(M)$ has standard form, that is, it is decomposed into the sum of a derivation and a center-valued trace.

Key words: *von Neumann algebra, measurable operator, type I von Neumann algebra, derivation, inner derivation, Lie derivation, center-valued trace.*

1. Introduction

The structure of Lie derivations on C^*-algebras, and on more general Banach algebras, has attracted some attention over the past years. Let A be an algebra over the field of complex numbers. A linear operator $D : A \to A$ is called a *derivation* if $D(xy) = D(x)y + xD(y)$ for all $x, y \in A$ (the Leibniz rule). Each element $a \in A$ defines a derivation D_a on A given by $D_a(x) = ax - xa$, $x \in A$. Such derivations D_a are said to be *inner derivations*. If the element a implementing the derivation D_a on A belongs to a larger algebra B containing A as a proper ideal, then D_a is called a *spatial derivation*. A linear operator $L : A \to A$ is called a *Lie derivation* if $L([x, y]) = [L(x), y] + [x, L(y)]$ for all $x, y \in A$, where $[x, y] = xy - yx$.

Let $Z(A)$ denote the center of A. A linear operator $\tau : A \to Z(A)$ is called a *center-valued trace* if $\tau(xy) = \tau(yx)$ for all $x, y \in A$. The problem of the standard decomposition for a Lie derivation in ring theory was studied in work by W. S. Martindale [8]. W. S. Martindale solved this problem for primitive rings containing nontrivial idempotents and with the characteristic not equal to 2. Following these results obtained for rings, C. R. Miers in [10] solved the problem of the standard decomposition for the case of von Neumann algebras. In [4], M. Brešar determinend the structure of Lie derivations of prime rings which does not satisfy the standard polynomial identity S_4. Banning and Mathieu [3] extended to semiprime rings the description of Lie derivations

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obtained by Bresar in the prime case. V. E. Johnson proved in [7] that every continuous Lie derivation L from a C^*-algebra A into a Banach A-bimodule X can be decomposed as $L = D + \tau$, where $D : A \to X$ is a derivation and τ is a center-valued trace from A into the center of X. This result was obtained by cohomological methods, namely the concept of symmetric amenability, and in fact holds for symmetrically amenable Banach algebras [7, Theorem 9.2]. In [2], P. Ara and M. Mathieu developed a theory of local multipliers of C^*-algebras which one deal with the situation of C^*-algebras which are non commutative enough. A positive answer has also recently been given for Lie derivations from an arbitrary C^*-algebra into itself [9] by combining the techniques of [2] and [13]. The present paper is devoted to the standard decomposition of Lie derivations on the algebra of measurable operators affiliated with a type I_n von Neumann algebra.

The present paper is devoted to the standard decomposition of Lie derivations on the algebra of measurable operators affiliated with a type I_n von Neumann algebra, and is a somewhat extended English version of [14].

2. Preliminaries

Throughout the paper, let H denote a Hilbert space, and let $B(H)$ be the algebra of all bounded linear operators acting on H. Let M denote a von Neumann subalgebra in $B(H)$, and let $P(M)$ be the complete lattice of all orthogonal projections in M.

A linear subspace $\mathcal{D}$ of H is said to be *affiliated with* M (written $\mathcal{D}\eta M$) if $u(\mathcal{D}) \subseteq \mathcal{D}$ for any unitary operator u belonging to the commutant

$$M' := \{y \in B(H) : xy = yx \text{ for all } x \in M\}$$

of the algebra M.

A linear operator x in H with domain $\mathcal{D}(x)$ is said to be *affiliated with* M (written $x\eta M$) if for each unitary operator $u \in M'$, $u(\mathcal{D}(x)) \subseteq \mathcal{D}(x)$ and $ux(\xi) = xu(\xi)$ for all $\xi \in \mathcal{D}(x)$.

A linear subspace $\mathcal{D}$ of H is said to be *strongly dense* in H with respect to the von Neumann algebra M if $\mathcal{D}\eta M$ and if there exists a sequence $\{p_n\}_{n=1}^{\infty}$ of projections in $P(M)$ such that $p_n \uparrow \mathbf{1}$, $p_n(H) \subset \mathcal{D}$ for each $n \in \mathbb{N}$, and $p_n^{\perp} = \mathbf{1} - p_n$ is a finite projection in M for each $n \in \mathbb{N}$; here, as subsequently, $\mathbf{1}$ stands for the unit of M.

A closed linear operator x acting in H is said to be *measurable* with respect to the von Neumann algebra M if $x\eta M$ and if $\mathcal{D}(x)$ is strongly dense in H. Throughout let $S(M)$ be the set of all measurable operators affiliated with M (see [12]) and let $Z(S(M))$ be the center of the algebra $S(M)$. A von Neumann algebra M is *of type I* if it contains a faithful abelian projection.

3. Structure of Lie Derivations

Let M be a homogeneous von Neumann algebra of type I_n ($n \in \mathbb{N}$), with the center Z. Then M is $*$-isomorphic to the algebra $M_n(Z)$ of $n \times n$ matrices over Z (see [11, Theorem 2.3.3]). In addition, $S(M) \cong M_n(Z(S(M)))$ and $Z(S(M)) = S(Z)$ (see [1]) as

$S(Z) \cong L^0(\Omega)$ (see [12]), where $L^0(\Omega) =: L^0$ is the algebra of all complex measurable functions on Ω, and $S(Z)$ is the algebra of measurable operators for the commutative von Neumann algebra Z. Moreover, any element $x \in M_n(S(Z))$ can be represented in the form $x = \sum\limits_{i,j=1}^{n} \lambda_{ij} e_{ij}$, where $\lambda_{ij} \in L^0$ and e_{ij} are the matrix units. Let $L : S(M) \to S(M)$ be any Lie derivation, and let $\psi = L|_{L^0}$ be the restriction of L to the center of L^0. This definition is correct because L maps the center into itself. Indeed, since, by definition, $L([z,x]) = [L(z), x] + [z, L(x)]$ for all $z \in L^0$ and all $x \in S(M)$, and since $[z, L(x)] = 0$ and $L([z,x]) = 0$, it follows that $[L(z), x] = 0$, i.e. $L(z)$ belongs to the center whenever $z \in L^0$.

Define $\tau(x) = \sum\limits_{i=1}^{n} \psi(\lambda_{ii})$ provided $x = \sum\limits_{i,j=1}^{n} \lambda_{ij} e_{ij}$.

Proposition. *τ is a linear map, and $\tau(xy) = \tau(yx)$.*

Proof. The linearity of L implies the linearity of τ. Let us prove that $\tau(xy) = \tau(yx)$. If we let

$$x = (\lambda_{ij}), \quad y = (\mu_{ij}), \quad xy = (c_{ij}), \quad c_{ii} = \sum_{k=1}^{n} \lambda_{ik}\mu_{ki},$$

$$yx = (b_{ij}), \quad b_{ii} = \sum_{k=1}^{n} \mu_{ik}\lambda_{ki}, \quad i,j = \overline{1,n},$$

then

$$\tau(xy) = \sum_{k=1}^{n} \psi(\lambda_{ik}\mu_{ki}) = \psi\left(\sum_{k=1}^{n} \lambda_{ik}\mu_{ki}\right) = \psi\left(\sum_{k=1}^{n} \mu_{ik}\lambda_{ki}\right) = \tau(yx).$$

Thus, $\tau : M_n(L^0) \to L^0$ is a center-valued trace. □

In order to prove the desired equality $L = D + \tau$, we shall show that $(L - \tau) = D$ is a derivation.

If $p_1 = p$ is a projection in $S(M)$, $\quad p_2 = \mathbf{1} - p$ then set $p_i S(M) p_j = \{p_i x p_j : x \in S(M)\}$ for $i,j = 1,2$. It is clear that $S(M) = \sum\limits_{i=1}^{2}\sum\limits_{j=1}^{2} p_i S(M) p_j$. Let further $M_{ij} = p_i S(M) p_j$ where $i,j = 1,2$, and recall that $M_{ij} \subset M_{ik} M_{kj}$ for $i,j = 1,2$.

Lemma 1. *Let p be a projection in $S(M)$. Then, for all $x \in S(M)$,*

$$\begin{aligned} x\{pL(p) + L(p)p + pL(p)p - L(p)\} - \{pL(p) + L(p)p + pL(p)p - L(p)\}x \\ = 3px\{pL(p) + L(p)p - L(p)\} - 3\{pL(p) + L(p)p - L(p)\}xp. \quad (1) \end{aligned}$$

Proof. The equality

$$[[[x,p],p],p] = [x,p] \tag{2}$$

holds for any $x \in S(M)$. Applying L to the identity (2), we obtain

$$L[[[x,p],p],p] = L[x,p],$$

$$\begin{aligned}
&[L([[x,p],p]),p]+[[[x,p],p],L(p)]=[[L([x,p]),p]+[[x,p],L(p)],p]+[[[x,p],p],L(p)]\\
&=[[[L(x),p]+[x,L(p)],p]+[[x,p],L(p)],p]+[[[x,p],p],L(p)]\\
&=[[L(x)p-pL(x)+xL(p)-L(p)x,p]+[xp-px,L(p)],p]+[[xp-px,p],L(p)]\\
&=[L(x)p-pL(x)p+xL(p)p-L(p)xp-pL(x)p+pL(x)-pxL(p)+pL(p)x\\
&+xpL(p)-pxL(p)-L(p)xp+L(p)px,p]+[xp-2pxp+px,L(p)]\\
&=L(x)p-pL(x)p+xL(p)p-L(p)xp-pL(x)p+pL(x)p\\
&-pxL(p)p+pL(p)xp+xpL(p)p-pxL(p)p-L(p)xp+L(p)pxp-pL(x)p\\
&+pL(x)p-pxL(p)p+pL(p)xp+pL(x)p-pL(x)+pxL(p)-pL(p)x\\
&-pxpL(p)+pxL(p)+pL(p)xp-pL(p)px+xpL(p)-2pxpL(p)+pxL(p)\\
&-L(p)xp+2L(p)pxp-L(p)px=L(x)p-pL(x)+xL(p)-L(p)x,
\end{aligned}$$

which implies the required equality. □

Lemma 2. *$L(p)=[p,s]+z$ for some $s\in S(M)$ and $z\in Z(S(M))$.*

Proof. Let $L(p)=\sum f_{ij}$, $f_{ij}\in M_{ij}$ $(i,j=1,2)$. Applying (1) for all $x\in S(M)$, we obtain

$$\begin{aligned}
&x\{p(f_{11}+f_{12}+f_{21}+f_{22})+(f_{11}+f_{12}+f_{21}+f_{22})p+p(f_{11}+f_{12}+f_{21}+f_{22})p\\
&-(f_{11}+f_{12}+f_{21}+f_{22})\}-\{p(f_{11}+f_{12}+f_{21}+f_{22})+(f_{11}+f_{12}+f_{21}+f_{22})p\\
&+p(f_{11}+f_{12}+f_{21}+f_{22})p-(f_{11}+f_{12}+f_{21}+f_{22})\}x\\
&=3px\{p(f_{11}+f_{12}+f_{21}+f_{22})+(f_{11}+f_{12}+f_{21}+f_{22})p-(f_{11}+f_{12}+f_{21}+f_{22})\}\\
&-3\{p(f_{11}+f_{12}+f_{21}+f_{22})+(f_{11}+f_{12}+f_{21}+f_{22})p-(f_{11}+f_{12}+f_{21}+f_{22})\}xp.
\end{aligned}$$

Since $f_{ij}\in M_{ij}$ $(i,j=1,2)$, it follows that $f_{ij}=p_if_{ij}p_j$. Therefore

$$\begin{aligned}
pf_{ij}&=pp_if_{ij}p_j=f_{ij}\ \text{ for } i=1, & pf_{ij}&=pp_if_{ij}p_j=0\ \text{ for } i\neq 1,\\
f_{ij}p&=p_if_{ij}p_jp=f_{ij}\ \text{ for } j=1, & f_{ij}p&=p_if_{ij}p_jp=0\ \text{ for } j\neq 1,\\
pf_{ij}p&=pp_if_{ij}p_jp=f_{ij}\ \text{ for } i=j=1, & pf_{ij}p&=pp_if_{ij}p_jp=0\ \text{ for } i,j\neq 1.
\end{aligned}$$

Thus

$$\begin{aligned}
p(f_{11}+f_{12}+f_{21}+f_{22})&=f_{11}+f_{12},\\
(f_{11}+f_{12}+f_{21}+f_{22})p&=f_{11}+f_{21},\\
p(f_{11}+f_{12}+f_{21}+f_{22})p&=f_{11},
\end{aligned}$$

$$\begin{aligned}
&x(f_{11}+f_{12}+f_{11}+f_{21}+f_{11}-f_{11}-f_{12}-f_{21}-f_{22})-(f_{11}+f_{12}+f_{11}+f_{21}+f_{11}\\
&-f_{11}-f_{12}-f_{21}-f_{22})x=3px(f_{11}+f_{12}+f_{11}+f_{21}-f_{11}-f_{12}-f_{21}-f_{22})\\
&-3(f_{11}+f_{12}+f_{11}+f_{21}-f_{11}-f_{12}-f_{21}-f_{22})xp.
\end{aligned}$$

Therefore

$$x(2f_{11}-f_{22})-(2f_{11}-f_{22})x=3px(f_{11}-f_{22})-3(f_{11}-f_{22})xp. \tag{3}$$

If $x \in M_{im}$ and $y \in M_{kj}$, then $xy = 0$ for $m \neq k$, and $xy \in M_{ij}$ for $m = k$.

If $x \in M_{12}$, then (3) implies $f_{11}x = xf_{22}$, whence it follows that

$$(f_{11} + f_{22})x = x(f_{11} + f_{22}) \qquad (x \in M_{12}),$$

because $f_{22}x = xf_{11} = 0$. Similarly, $(f_{11} + f_{22})x = x(f_{11} + f_{22})$ $(x \in M_{21})$. Now let $x \in M_{11}$ and $y \in M_{12}$. Then

$$\begin{aligned}\{(f_{11} + f_{22})x - x(f_{11} + f_{22})\}\, y &= (f_{11} + f_{22})xy - x(f_{11} + f_{22})y \\ &= (f_{11} + f_{22})xy - xy(f_{11} + f_{22}) = (f_{11} + f_{22})xy - (f_{11} + f_{22})xy = 0,\end{aligned}$$

because $y, xy \in M_{12}$. It follows that

$$\{(f_{11} + f_{22})x - x(f_{11} + f_{22})\}\, y = 0$$

for all $y \in M_{12}$. From here, we obtain

$$(f_{11} + f_{22})x - x(f_{11} + f_{22}) = 0 \qquad (x \in M_{11}).$$

Similarly,

$$(f_{11} + f_{22})x - x(f_{11} + f_{22}) = 0 \qquad (x \in M_{22}),$$

i.e. $f_{11} + f_{22} = z \in Z(S(M))$. Hence, $L(p) = (f_{12} + f_{21}) + z$ and, setting $s = f_{12} - f_{21}$, we obtain $L(p) = (ps - sp) + z$. □

Throughout the rest of this paper we impose the additional assumption that $L(p)$ is an element of $Z(S(M))$.

Lemma 3. *$L(M_{ij}) \subset M_{ij}$ if $i \neq j$.*

Proof. Let $x \in M_{12}$ and $L(x) = \sum y_{ij}$ where $y_{ij} \in M_{ij}$ for $i, j = 1, 2$. Then, taking into account the equality $x = [p, x]$, we obtain

$$\sum y_{ij} = L(x) = L([p, x]) = [L(p), x] + [p, L(x)] = [p, L(x)] = y_{12} - y_{21},$$

since $L(p) \in Z(S(M))$. It follows that $y_{11} = y_{21} = y_{22} = 0$. Thus, $L(x) \in M_{12}$. The case of $x \in M_{21}$ can be proved similarly. □

Lemma 4. *$D(M_{ii}) \subset M_{ii}$.*

Proof. Let $x \in M_{11}$ and $L(x) = \sum y_{ij}$, $y_{ij} \in M_{ij}$. Then $[p, x] = 0$ and $0 = L([p, x]) = [L(p), x] + [p, L(x)] = y_{12} - y_{21}$, and so $y_{12} = y_{21} = 0$ and $L(x) \in M_{11} + M_{22}$. Similarly, if $x \in M_{22}$, then $L(x) \in M_{11} + M_{22}$. Let $x \in M_{11}$ and $y \in M_{22}$, and let $L(x) = a_{11} + a_{22}$ and $L(y) = b_{11} + b_{22}$ where $a_{ii}, b_{ii} \in M_{ii}$. Then $0 = L([x, y]) = [L(x), y] + [x, L(y)] = [a_{22}, y] + [x, b_{11}] = 0$, where $[a_{22}, y] \in M_{22}$ and $[x, b_{11}] \in M_{11}$. Hence, in particular, $[a_{22}, y] = 0$ for all $y \in M_{22}$, i.e. a_{22} is a central element in M_{22}, and so $a_{22} = (\mathbf{1} - p)z$, $z \in S(Z)$. Therefore

$$L(x) = a_{11} + (\mathbf{1} - p)z = [(a_{11} - pz) + z] \in M_{11} + S(Z),$$

where $z \in S(Z)$.

On the other hand, $L = D + \tau$, and hence $L(x) = D(x) + z$ for some $z \in S(Z)$. Comparing the last equalities gives $D(x) \in M_{11}$ where $x \in M_{11}$. A similar argument holds if $x \in M_{22}$. □

Now we prove that $L-\tau=D$ is a derivation on elements of M_{ij}.

Lemma 5. *$D(xy)=D(x)y+xD(y)$ for $x\in M_{ii}$ and $y\in M_{jk}$ $(j\neq k)$.*

Proof. Let $x\in M_{11}$ and $y\in M_{12}$. Then $D(xy)=L(xy)-\tau(xy)=L(xy),\tau(xy)=\tau(yx)=0$, since $yx=0$. Therefore $D(xy)=L[x,y]=[L(x),y]+[x,L(y)]=[(D+\tau)(x),y]+[x,(D+\tau)(y)]=[D(x),y]+[x,D(y)]$, because $\tau(x),\tau(y)$ are central elements fulfilling $[\tau(x),y]=[x,\tau(y)]=0$. It follows that $D(xy)=D(x)y+xD(y)$, since $yD(x)=D(y)x=0$. The case in which $x\in M_{22}$ and $y\in M_{21}$ can be proved similarly. □

Lemma 6. *$D(xy)=D(x)y+xD(y)$ for $x\in M_{ii}$ and $y\in M_{jj}$.*

Proof. Let $x,y\in M_{11}$, $r\in M_{12}$, then by Lemma 5 we have

$$D((xy)r)=D(xy)r+xyD(r).$$

$$\begin{aligned}D(xy)r&=D(xyr)-xyD(r)=D(x)yr+xD(yr)-xyD(r)\\&=D(x)yr+x\left\{D(y)r+yD(r)\right\}-xyD(r)=\left\{D(x)y+xD(y)\right\}r.\end{aligned}$$

Hence, $\left\{D(xy)-D(x)y-xD(y)\right\}r=0$ for all $r\in M_{12}$. It follows that $D(xy)-D(x)y-xD(y)=0$. The case when $x\in M_{22}$ and $y\in M_{22}$ can be proved similarly. □

Theorem 1. *D is a derivation from $S(M)$ into $S(M)$.*

Proof. We have to prove that $D(xy)=D(x)y+xD(y)$ for all $x,y\in S(M)$. Let $x\neq 0\in M_{12}$, $y\in M_{21}$. Then the equality

$$\begin{aligned}\tau([x,y])&=L([x,y])-D([x,y])=[L(x),y]+[x,L(y)]-D([x,y])\\&=[D(x),y]+[x,D(y)]-D(xy)+D(yx)\end{aligned}$$

implies $\{D(x)y+xD(y)-D(xy)\}+\{D(yx)-D(y)x-yD(x)\}=0$. Therefore $[D(x)y+xD(y)-D(xy)]\in(M_{11}\cap M_{22})$, i.e. $[D(x)y+xD(y)-D(xy)]=0$. □

Corollary. *If $L|_{L^0}=0$, then any Lie derivation from $S(M)$ into $S(M)$ is a derivation.*

Remark. We supposed in the proof of Lemma 3 that $L(p)\in S(Z)$. In reality, according to Lemma 2 one can write $L(p)=[p,s]+z$, where $p\in S(M)$, $z\in S(Z)$. An element $s\in S(M)$ defines the inner derivation D_s by the rule: $D_s(x)=sx-xs$ for all $x\in S(M)$. Consider the Lie derivation $L'=L-D_s$ from $S(M)$ into $S(M)$. It is clear that $L'(p)=z\in S(Z)$. By Theorem 1, $L'=D+\tau$ or $L=(D+D_s)+\tau$.

Our standard decomposition result now follows from Theorem 1 and its Corollary, and we state it as the following main theorem.

Theorem 2. *Let M be a homogeneous von Neumann algebra of type I_n. Any Lie derivation on $S(M)$ can be uniquely represented in the form*

$$L=D+\tau,$$

where D is a derivation and τ is a center-valued trace from $S(M)$ into $S(Z)$.

Another proof of Theorem 2, not essentially distinct from that given above, but different in form and detail, is offered in [14].

Let A be a commutative algebra, and let $M_n(A)$ be the algebra of $n \times n$ matrices over A. If e_{ij} $\left(i, j = \overline{1, n}\right)$ are the matrix units in $M_n(A)$, then each element $x \in M_n(A)$ has form

$$x = \sum_{i,j=1}^{n} \lambda_{ij} e_{ij}, \quad \text{where } \lambda_{ij} \in A, \quad i, j = \overline{1, n}.$$

Let $\delta : A \to A$ be a derivation. Setting

$$D_\delta \left(\sum_{i,j=1}^{n} \lambda_{ij} e_{ij} \right) = \sum_{i,j=1}^{n} \delta(\lambda_{ij}) e_{ij}, \tag{4}$$

we obtain a well-defined linear operator D_δ on the algebra $M_n(A)$. Moreover, D_δ is a derivation on the algebra $M_n(A)$ and its restriction onto the center of the algebra $M_n(A)$ coincides with the given δ. Now Lemma 2.2 [1] implies the following.

Corollary. *Let M be a homogeneous von Neumann algebra of type I_n, $n \in \mathbb{N}$. Every Lie derivation L on the algebra $S(M)$ can be uniquely represented as a sum $L = D_a + D_\delta + \tau$, where D_a is an inner derivation implemented by an element $a \in S(M)$, while D_δ is the derivation of the form (4) generated by a derivation δ on the center $S(M)$ identified with $S(Z)$.*

Now let M be an arbitrary finite von Neumann algebra of type I with center Z. There exists a family $\{z_n\}_{n \in F}$ $(F \subseteq \mathbb{N})$ of central projections from M with $\sup_{n \in F} z_n = \mathbf{1}$ such that the algebra M is $*$-isomorphic with the C^*-sum of von Neumann algebras $z_n M$ of type I_n $(n \in F)$, i.e.

$$M \cong \bigoplus_{n \in F} z_n M.$$

By Proposition 1.1 [1] we have that

$$S(M) \cong \prod_{n \in F} S(z_n M).$$

Suppose that D is a derivation on $S(M)$, and that δ is its restriction onto its center $S(Z)$. Since δ maps each $z_n S(Z) \cong Z(S(z_n M))$ into itself, δ generates a derivation δ_n on $z_n S(Z)$ for each $n \in F$. Let D_{δ_n} be the derivation on the matrix algebra $M_n(z_n Z(S(M))) \cong S(z_n M)$ defined as in (4). Put

$$D_\delta(\{x_n\}_{n \in F}) = \{D_{\delta_n}(x_n)\}, \quad \{x_n\}_{n \in F} \in S(M). \tag{5}$$

Then the map D_δ is a derivation on $S(M)$. Now Lemma 2.3 [1] implies the following.

Corollary. *Let M be a finite von Neumann algebra of type I. Every Lie derivation L on the algebra $S(M)$ can be uniquely represented as a sum $L = D_a + D_\delta + \tau$, where D_a is an inner derivation implemented by an element $a \in S(M)$, and D_δ is a derivation given as in (5).*

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АННОТАЦИЯ

Для алгебр измеримых операторов, присоединенных к конечной алгебре фон Неймана типа I, доказана теорема о представлении лиевых дифференцирований в виде суммы ассоциативного дифференцирования и центрозначного следа.
Ключевые слова: *алгебра фон Неймана, измеримый оператор, алгебра фон Неймана типа I, дифференцирование, лиево дифференцирование, внутреннее дифференцирование, центрозначный след.*